Monday, October 12, 2015

p. 494: 1, 3, 4, 5, 6, 7

Problem 1

Problem. Find the center of mass of the point masses $m_1 = 7$, $m_2 = 3$, and $m_3 = 5$ lying at $x_1 = -5$, $x_2 = 0$, and $x_3 = 3$, respectively, on the x-axis.

Solution. The numerator is

$$\begin{aligned} m_1x_1 + m_2x_2 + m_3x_3 &= -35 + 0 + 15 \\ &= -20. \end{aligned}$$

The denominator is

$$\begin{aligned} m_1 + m_2 + m_3 &= 7 + 3 + 5 \\ &= 15. \end{aligned}$$

The center of mass is

$$\begin{aligned} \overline{x} &= \frac{-20}{15} \\ &= -\frac{4}{3}. \end{aligned}$$

Problem 3

Problem. Find the center of mass of the point masses $m_1 = 1$, $m_2 = 3$, $m_3 = 2$, $m_4 = 9$, and $m_5 = 5$ lying at $x_1 = 6$, $x_2 = 10$, $x_3 = 3$, $x_4 = 2$, and $x_5 = 4$, respectively, on the x-axis.

Solution. The numerator is

$$\begin{aligned} m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5x_5 &= 6 + 30 + 6 + 18 + 20 \\ &= 80. \end{aligned}$$

The denominator is

$$\begin{aligned} m_1 + m_2 + m_3 + m_4 + m_5 &= 1 + 3 + 2 + 9 + 5 \\ &= 20. \end{aligned}$$

The center of mass is

$$\begin{aligned}\overline{x} &= \frac{80}{20} \\ &= 4.\end{aligned}$$

Problem 4

Problem. Find the center of mass of the point masses $m_1 = 8$, $m_2 = 5$, $m_3 = 5$, $m_4 = 12$, and $m_5 = 2$ lying at $x_1 = -2$, $x_2 = 6$, $x_3 = 0$, $x_4 = 3$, and $x_5 = -5$, respectively, on the x-axis.

Solution. The numerator is

$$\begin{aligned}m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5x_5 &= -16 + 30 + 0 + 36 - 10 \\ &= 40.\end{aligned}$$

The denominator is

$$\begin{aligned}m_1 + m_2 + m_3 + m_4 + m_5 &= 8 + 5 + 5 + 12 + 2 \\ &= 32.\end{aligned}$$

The center of mass is

$$\begin{aligned}\overline{x} &= \frac{-40}{32} \\ &= \frac{5}{4}.\end{aligned}$$

Problem 5

Problem. (a) Translate each point mass in Exercise 3 to the right four units and determine the resulting center of mass.

(b) Translate each point mass in Exercise 4 to the left two units and determine the resulting center of mass.

Solution. (a) The new positions are $x_1 = 10$, $x_2 = 14$, $x_3 = 7$, $x_4 = 6$, and $x_5 = 8$. The numerator is

$$\begin{aligned}m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5x_5 &= 10 + 42 + 14 + 54 + 40 \\ &= 160.\end{aligned}$$

The denominator is still

$$\begin{aligned} m_1 + m_2 + m_3 + m_4 + m_5 &= 1 + 3 + 2 + 9 + 5 \\ &= 20. \end{aligned}$$

The center of mass is

$$\begin{aligned} \overline{x} &= \frac{160}{20} \\ &= 8. \end{aligned}$$

(b) The new positions are $x_1 = -4$, $x_2 = 4$, $x_3 = -2$, $x_4 = 1$, and $x_5 = -7$. The numerator is

$$\begin{aligned} m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5x_5 &= -32 + 20 - 10 + 12 - 14 \\ &= -24. \end{aligned}$$

The denominator is

$$\begin{aligned} m_1 + m_2 + m_3 + m_4 + m_5 &= 8 + 5 + 5 + 12 + 2 \\ &= 32. \end{aligned}$$

The center of mass is

$$\begin{aligned} \overline{x} &= \frac{-24}{32} \\ &= -\frac{3}{4}. \end{aligned}$$

Problem 6

Problem. Use the result of Exercise 5 to make a conjecture about the change in the center of mass that results when each point mass is translated k units horizontally.

Solution. The new center of mass will be at $\overline{x} + k$.

Problem 7

Problem. Consider a beam of length 10 with a fulcrum x feet from on end. There are two children with weights 48 and 72, respectively, placed on opposite ends of the beam. Find x such that the system is in equilibrium.

Solution. We find $48 + 72 = 120$, so the 48-lb child should be $\frac{72}{120}$, or $\frac{3}{5}$ of the way from x and the 72-lb child should be $\frac{48}{120}$, or $\frac{2}{5}$ from x, so that the products $48 \times \frac{72}{120}$ and $72 \times \frac{48}{120}$ will be equal.

Since the board is 10 feet long, the fulcrum should be placed 4 feet from the heavier child and 6 feet from the lighter child.